Dynamic critical phenomena and real-time functional renormalization group

Yuya Tanizaki

Department of Physics, The University of Tokyo

Theoretical Research Division, Nishina Center, RIKEN

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Collaborators: David Mesterházy (Bern), Jan H. Stockemar (Heidelberg)

Introduction

- We would like to understand dissipative dynamics from the viewpoint of microscopic QFT.
- Dynamic universality classes are much more complicated than static ones.
- Real-time functional RG is a new possibility to treat dynamic fluctuations in a systematic manner.

Effective description of model A

Purely dissipative relaxation: Langevin eq.
(ϕ: order para., $\mathcal{H}$: Landau-Ginzburg Hamiltonian, η: random force)

$$\partial_t \phi = -D \frac{\delta \mathcal{H}}{\delta \phi} + \eta.$$

Assumption: Time scale of $\phi \gg$ Microscopic time scale

Random force is a white Gaussian noise with Einstein's rel.

$$\langle \eta(t,x)\eta(t',x')\rangle = \frac{2D}{\beta}\delta(t-t')\delta(x-x').$$

Equivalent field theory

Transition amplitude: ($\tilde{\phi}$: Martin-Siggia-Rose response field)

$$\begin{aligned} Z &= \int \mathcal{D}\phi \mathcal{D}\tilde{\phi} \mathcal{D}\eta \exp\left[\mathrm{i}\tilde{\phi}\left(\partial_t \phi + D\frac{\delta \mathcal{H}}{\delta \phi} - \eta\right) - \frac{\beta}{4D}\eta^2\right] \\ &= \int \mathcal{D}\phi \mathcal{D}\tilde{\phi} \exp\left[\mathrm{i}\tilde{\phi}\left(\partial_t \phi + D\frac{\delta \mathcal{H}}{\delta \phi}\right) - \frac{D}{\beta}\tilde{\phi}^2\right] \end{aligned}$$

Question

Can one construct this theory starting form microscopic QFT?

(See Morimatsu et. al. (arXiv:1411.1867) which applies the 2PI method for this purpose.)

Goals

- We develop the renormalization group for CTP effective actions: Real-time FRG.
- We compute the dynamic critical phenomenon of the relativistic QFT using the real-time FRG. The microscopic action is

$$S[\varphi]=\int \mathrm{d}t\,\mathrm{d}^d\boldsymbol{x}\ \left(\frac{1}{2}(\partial_\mu\varphi_a)^2-\frac{m^2}{2}\varphi_a^2-\frac{\lambda}{4}(\varphi_a^2)^2\right)$$

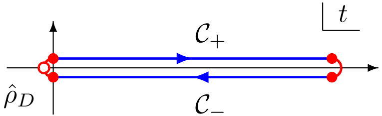
Closed-time path formalism & Fluctuation-dissipation theorem

Closed-time path (CTP) formalism

Schwinger–Keldysh formalism:

$$
\begin{aligned}
Z &= \mathrm{tr}\left[\hat{\rho}_D(t_f,t_i)\right] = \mathrm{tr}\left[\mathrm{e}^{-\mathrm{i}\hat{H}(t_f-t_i)}\hat{\rho}_D \mathrm{e}^{\mathrm{i}\hat{H}(t_f-t_i)}\right] \\
&= \int \mathcal{D}\varphi_+\mathcal{D}\varphi_- \langle \varphi_+; t_i|\hat{\rho}_D|\varphi_-;t_i\rangle \exp\mathrm{i}\left(S[\varphi_+]-S[\varphi_-]\right).
\end{aligned}
$$

We need forward and backward paths along the real-time direction:



Classical and quantum fields

Introduce classical and quantum fields

$$\varphi = (\varphi_+ + \varphi_-)/2,\ \tilde{\varphi} = \varphi_+ - \varphi_-.$$

Motivation

φ becomes the order parameter field, and $\tilde{\varphi}$ the MSR field:

$$S[\varphi_+] - S[\varphi_-] = \int_{t,x} \tilde{\varphi}(t,x) \frac{\delta S[\varphi]}{\delta \varphi(t,x)} + O(\tilde{\varphi}^3).$$

The leading term gives the classical eom.

Fluctuation-Dissipation Theorem

Retarded and statistical propagators are closely related.

$$\langle\varphi(p)\varphi(-p)\rangle = \coth\frac{\beta p^0}{2}\mathrm{Im}\big(\mathrm{i}\langle\varphi(p)\tilde{\varphi}(-p)\rangle\big).$$

Take the derivative expansion of the inverse retarded propagator.

$$\Gamma^{(2\mathrm{pt})}(p) = \begin{pmatrix} 0 & Z^{\parallel}\omega^2 - Z^{\perp}\boldsymbol{p}^2 + \mathrm{i}\Omega\omega \\ Z^{\parallel}\omega^2 - Z^{\perp}\boldsymbol{p}^2 - \mathrm{i}\Omega\omega & \mathrm{i}\Omega\omega\coth(\beta\omega/2) \end{pmatrix}.$$

The same Ω must appear thanks to FDT!

Local interaction approximation

The interaction part of the CTP effective action is assumed to be local,

$$\mathcal{U} = m^2\sigma_2 + \lambda_{1,2}\left(\sigma_1 - v^2/2\right)\sigma_2 + \lambda_{2,3}\sigma_2\sigma_3,$$

with the $O(N)$ invariants,

$$\sigma_1 = \frac{1}{2}\phi^a\phi_a, \quad \sigma_2 = \phi^a\tilde{\phi}_a, \quad \sigma_3 = \frac{1}{2}\tilde{\phi}^a\tilde{\phi}_a,$$

Local interaction approximation

Another term, such as

$$\mathrm{i}\lambda_{1,3}\sigma_1\sigma_3,$$

may be included as a local interaction from the viewpoint of symmetry, but FDT states that

$$\lambda_{1,3} = 0.$$

In the UV region (or $T = 0$), furthermore, FDT gives another constraint:

$$\lambda_{1,2} = 4\lambda_{2,3} = \lambda/3.$$

Real-time functional renormalization group

Wetterich equation

Schwinger functional W_Λ with an IR regulator R_Λ:

$$\exp(W_\Lambda[J]) = \int \mathcal{D}\phi \exp\left(-S[\phi] - \frac{1}{2}\phi \cdot R_\Lambda \cdot \phi + J \cdot \phi\right).$$

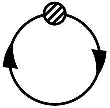
The 1PI effective action $\Gamma_\Lambda$is introduced via the Legendre trans.:

$$\Gamma_\Lambda[\varphi] + \frac{1}{2}\varphi \cdot R_\Lambda \cdot \varphi = J[\varphi] \cdot \varphi - W_\Lambda[J[\varphi]],$$

which obeys the flow equation (Wetterich 1993, Ellwanger 1994, Morris 1994)

$\partial_\Lambda R_\Lambda$

$$\partial_\Lambda \Gamma_\Lambda = -\partial_\Lambda W_\Lambda = \frac{1}{2}$$



$$[\delta^2\Gamma_\Lambda/\delta\varphi\delta\varphi + R_\Lambda]^{-1}$$

Properties of Γ_Λ: $\Gamma_\Lambda \to S$ as $R_\Lambda \to \infty$, and $\Gamma_\Lambda \to \Gamma$ as $R_\Lambda \to 0$.

Functional Renormalization Group

FRG modifies the CTP action as $S[\varphi,\tilde{\varphi}]\to S[\varphi,\tilde{\varphi}]+\Delta_k S[\varphi,\tilde{\varphi}]$, where

$$\Delta_k S[\varphi,\tilde{\varphi}]=-\int_{x,y}\tilde{\varphi}^a(x)R_{k,ab}(x,y)\varphi^b(y).$$

The function R_k is a momentum-dependent mass term, i.e.

$$R_{k,ab}(x^0,y^0;\boldsymbol{p})=R_k(\boldsymbol{p})\delta(x^0-y^0)\delta_{ab},$$

The initial density matrix must also be modified to respect FDT.

Renormalization Group equation

The 1PI effective action Γ_k follows the one-loop exact RG equation:

$$\frac{\partial}{\partial s}\Gamma_k = i\int_p \mathrm{Tr}\left\{\frac{\partial}{\partial s}R_k(\boldsymbol{p})\ \mathrm{Re}G_k^{\mathrm{R}}(\omega,\boldsymbol{p})\right\},$$

with $s=\ln k/\Lambda$. It connects quantum and classical effective actions:

$$\Gamma_k\to\Gamma \quad \text{as} \quad s\to-\infty, \qquad \Gamma_k\to S \quad \text{as} \quad s\to 0.$$

Scaling properties

At low energies and momenta, we assume the scaling property

$$\omega \sim |\boldsymbol{p}|^z.$$

Scale-invariance of the lowest order derivatives tells the scaling dimensions of classical/quantum fields:

$$[\varphi] = \frac{1}{2}(d - 2 + \eta^{\perp}),\ [\tilde{\varphi}] = [\varphi] + z.$$

The scaling dim. of quantum fields is larger than that of classical fields by z.

$$\begin{array}{rl} z = 1 & \text{UV fixed point} \\ z = 2 + c\eta^{\perp} & \text{IR fixed point} \end{array}$$

(Mesterházy, Stockemer, YT, PRD 92 (2015) 076001)

Dispersion Relation

Dispersion relation at high-temperature phase of the order-parameter field φ:

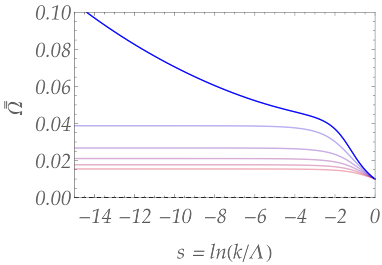
$$\omega(\boldsymbol{p}) \simeq \mathrm{i}\,\frac{Z^{\perp}}{\Omega T}(m_R^2 + \boldsymbol{p}^2).$$

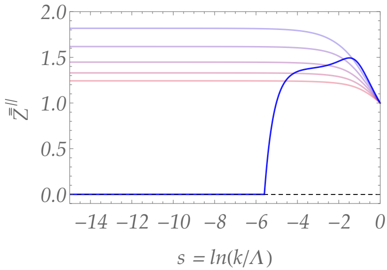
Microscopic QFT and the macroscopic dynamics (**model A**) is connected within our *ansatz*.

Numerical results

RG flow at the critical temperature $T_{\Lambda,\mathrm{cr}}$

($N = 1$ at $T \neq 0$.) (Mesterházy, Stockemer, YT, PRD 92 (2015) 076001)



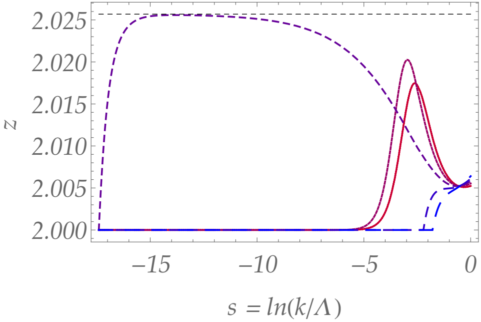


We obtained the scaling behavior:

$$\Omega \sim \mathrm{e}^{-s\eta_\Omega},\ Z^{||} = 0.$$

As the RG scale is lowered, $Z^{||}$ goes inside the negative region. This behaviors is out of the scope of our truncation, and we put $Z^{||} = 0$ at low-energy scales.

Dynamic scaling exponent



We find the dynamic scaling exponent is (Meserházy, Stockemer, YT, PRD 92 (2015) 076001)

$$\mathbf{z} \simeq \mathbf{2.025}$$

This is almost consistent with previous studies of model A.

Monte Carlo (Physica A 214 (1995) 547, JPSJ 69 (2000) 1931), FRG for Langevin eq. (arXiv:cond-mat/0606530)

ε-expansion

Expansion around the upper critical dimension

Set $d = 4 - \varepsilon$. One can check the regulator dependence of static and dynamic scaling exponents (Mesterházy, Stockemer, YT, PRD 92 (2015) 076001):

	$\eta^\perp / \left(\frac{N+2}{(N+8)^2}\varepsilon^2\right)$	$(z-2)/\eta^\perp$
Exponential cutoff[1]	$1/2$	0.73
Litim cutoff[2]	$1/2$	$1/2$
Sharp cutoff[3]	∞	-1
Effective theory	$1/2$	0.73

Cutoff functions are given by

$$R_k(\boldsymbol{p}) = Z^\perp \boldsymbol{p}^2 r(\boldsymbol{p}^2/k^2),$$

where (^{1}) $r_{\text{exp}} = (e^y - 1)^{-1}$, (^{2}) $r_{\text{opt}} = (1/y - 1)\,\theta(1-y)$, (^{3}) $r_{\text{sharp}} = 1/\theta(y-1) - 1$.

Summary

Summary

- Fluctuation-dissipation theorem plays an important role to introduce the dissipative dynamics.
- Our low-energy description gives the model A, and the critical exponents are consistent with previous studies.
- However, the obtained model A dynamics is a consequence of our ansatz.

Questions

Question

How can we describe model H?

There are several conserved quantities:

- Energy-momentum tensor: $T^{0\mu}$
- Noether charge (Baryon number): j^0

Can we treat the mode coupling among φ and these composite fields starting from microscopic QFT?